

## **Development of a Simulation-based Optimisation Environment for a Capacitated Multi-Echelon Production-Inventory System**

### ***Entwicklung einer simulationsbasierten Optimierungsumgebung für ein mehrstufiges kapazitiertes Produktionsbestandssystem***

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**Abstract:** One of the most important aspects affecting the performance of a supply chain is the management of inventories. Managing inventory in complex supply chains is typically difficult, and may have a significant impact on the customer service level and system-wide cost. In this paper, we present an efficient way using simulation-based optimisation approach for optimal inventory control of a multi-echelon production-inventory system under the continuous review policy. The Pareto dominance concept is implemented to find a set of near optimal solutions for determining the best trade-off between objectives. The Multiobjective Particle Swarm Optimisation (MOPSO) algorithm is used to determine the appropriate inventory control parameters to minimize the total inventory cost and maximize the service level. We have built an object-oriented simulation tool to evaluate the control parameters generated by the MOPSO.

## **1 Introduction**

Today, most companies source globally, produce in various plants and serve customers all over the world with a complex distribution network which has several stock points linked by various activities. Globalization of supply chains brings some challenges as well as benefits. If an item moves through more than one step before reaching the final customer, the supply chain is called multi-echelon supply chain. To meet new challenges, Supply Chain (SC) members must focus on the coordination and control of the multi-echelon system. Due to the growing complexity of these networks, efficient and effective inventory management in the supply chain become more important in order to improve the customer service level and reduce the whole system cost. Inventory cost is one of the major logistics costs that need to be optimized in any logistic system. As mentioned in Silver and Peterson (1985), 34% of the current assets and 90% of the working capital of a

typical company in USA are tied up in inventories. However, managing inventories in a dynamic environment and under uncertainty is typically difficult. It is easy to see that almost every inventory problem involves multiple and conflicting objectives which need to be optimized simultaneously such as maximizing the profit of each participant enterprise and minimizing the total cost of the system. Many researchers have dedicated themselves to deal with inventory problems involving more than one objective over the past several decades. The role of the inventory management is to try to find a way to minimize the level of their stocks without negatively impacting availability or customer service levels.

In the past, classical analytical techniques using mathematical programming methods such as probability theory have been used to obtain the approximate solution of multi-echelon inventory systems. Although analytical models are useful in many cases, they are not able to efficiently deal with the uncertainty and SC dynamics because of the inability of representing stochastic behaviours or highly complex relations between the different entities existing in real-world problems (Mele et al. 2006). Unlike traditional mathematical models, simulation is a powerful computer-based tool that enables to model the complex and uncertain situations of the real-world problem without the limiting assumptions (Banks, 2000).

Although computer simulation technology has been widely used as one of the most powerful technologies for analysing and improving enterprises' supply chain and logistics operations, simulation provides no concrete solutions to optimisation problems. Since users need to evaluate many feasible solutions in order to find a good solution to the problem, they may require large amounts of development and running time, which typically makes them inadequate for solving problems (Keskin et al. 2010). In the simulation-based optimisation approach, optimisation techniques are integrated into the simulation analysis. In mathematical programming the decision variables are assigned in an analytical function of decision variables. This function is known as the objective function. In the simulation-based optimisation, the performance measure becomes one (or a function of several) of the responses generated by a simulation model (Ammerie et al. 2010; Mele et al. 2006). The goal of simulation-based optimisation (SBO) is to find the best solution variables among many sets of model specifications (e.g. selected warehouse, allocation of customers, inventory control parameters) that lead to the optimal performance without explicitly evaluating each possibility (Carson & Maria, 1997).

In recent years, metaheuristic algorithms such as, Evolutionary Computation (EC), Simulated Annealing (SA), Tabu Search (TS), Particle Swarm Optimisation (PSO), and others, have been applied to various optimisation problems as a successful alternative to classical approaches (Silvia et al. 2003; Altıparmak et al. 2006). An evolutionary computation to find the non-dominated solutions for stochastic multiobjective (R,Q) inventory control systems is presented by Tsou (2009). He proposed two evolutionary optimizers, multiobjective electromagnetism-like optimisation (MOEMO) and multiobjective particle swarm optimisation (MOPSO). Computational results show that the evolutionary Pareto optimizers are fast algorithms that generate the non-dominated policies in term of lot size and safety stock. Hiremath et al. (2010) applied enhanced particle swarm optimisation (EPSO) to solve a three stage inventory problem to address the outsourcing issues with different shipment policies. A Particle Swarm Optimisation algorithm is proposed to maintain the optimal stock levels in the supply chain by Narmadha et al. (2010).

Their proposed methodology reduced the total supply chain cost as it undoubtedly established the most probable excess stock level and shortage level along with the consideration of lead time in supplying the stocks. Silva and Choello (2007) developed an optimisation model of a simplified supply chain based on PSO, including stocks, production, transportation and distribution, in an integrated production-inventory-distribution system.

Our study differs from those listed above in some ways such as varieties of complex factors, stochastic customer demand, different customer arrival distributions, the distance and the quantity dependent transportation cost, stochastic lead time, and the limited manufacture producing capacity, etc. The primary objective of this research is the development of a simulation-based optimisation tool for a multi-echelon capacitated production-inventory system to find the optimal inventory policies for each facility in a supply chain under a stochastic environment. A brief description of Particle Swarm Optimisation is presented in Section 2. The detail of the proposed simulation-based optimisation approach is given in Section 3. Section 4 discusses the case study and computational results. Finally, Section 5 concludes the current work and provides directions for the further research.

## 2 Particle Swarm Optimisation (PSO) Overview

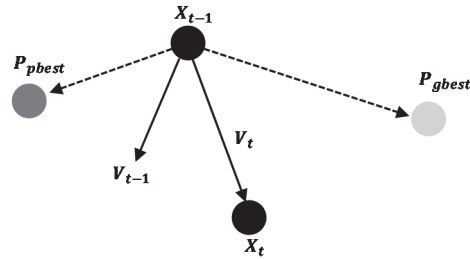
Particle swarm optimisation (PSO) is a stochastic optimisation technique based on population inspired by social behaviour (Kennedy & Eberhart, Particle swarm optimization, 1995). The PSO algorithm consists of a swarm of particles representing a solution point in a multidimensional, real valued search space of possible solutions. Each particle maintains its position in the search space with a velocity under the influence of the best solution found so far by each particle (the personal best) and the best solution found so far by the whole swarm (the global best) (Fig. 1). The last part considered in the PSO algorithm is the inertia part that is the memory of its previous velocity. In the  $n$ -th dimension of the search space, the calculation of each particle's velocity ( $V^n$ ) and position ( $X^n$ ) is achieved by following formulae:

$$V_t^n = w \times V_{t-1}^n + C_1 \times rand_1 \times (P_{pbest}^n - X_{t-1}^n) + C_2 \times rand_2 \times (P_{gbest}^n - X_{t-1}^n) \quad (1)$$

$$X_t^n = X_{t-1}^n + V_t^n \quad (2)$$

where

$n$	number of elements in a particle,
$w$	inertia weight of the particle,
$t$	generation number,
$C_1, C_2$	acceleration constants,
$rand$	random value between 0 and 1
$P_{pbest}^n$	local best position of the particle,
$P_{gbest}^n$	global best position of particle in the swarm.



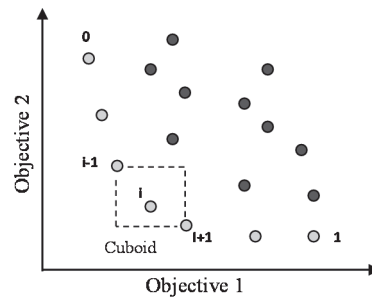
$X_{t-1}$ : current position,  $X_t$ : modified position,  $V_{t-1}$ : current velocity,  $V_t$ : modified velocity,

$P_{pbest}$ : local best position,  $P_{gbest}$ : global best position

**Figure 1:** Concept of modification of a searching point by PSO

## 2.1 Multiobjective Particle Swarm Optimisation (MOPSO)

Multiobjective problems are very complex and quite hard to solve by conventional optimisation techniques. In order to apply the PSO strategy for solving multiobjective optimisation problems, it is obvious that the original scheme has to be modified. Most multiobjective PSO works with the domination concept. There usually exists a set of solutions for the multiple objective cases which cannot simply be compared with each other. The non-dominated solutions are defined as a solution which dominates the others but do not dominate themselves. A solution  $X$  in the objective space is called Pareto-optimal (non-dominated solution), if and only if there is no other solution  $Y$  in the search space which could dominate  $X$ . In other words,  $X$  dominates  $Y$ , if  $X$  is better than  $Y$  in at least one objective function and not worse with respect to all other objective functions (Yu & Gen, 2010). The set including all Pareto-optimal solutions is termed the Pareto set.



**Figure 2:** Crowding distance calculation

One of the successful applications of PSO to multiobjective problems was proposed by Sierra and Coello Coello (2005), which is based on Pareto dominance and the use of a crowding factor to determine the direction for each particle. Compared with the original PSO, multiobjective PSO uses a set of leaders which is usually stored in a

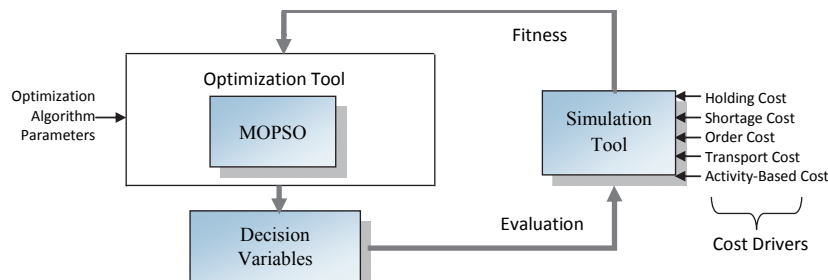
different place from the swarm. The leaders archive includes the best non-dominated solutions found since the beginning of the optimisation. At each generation, the global best solution is randomly selected using a roulette wheel selection in the leaders archive and a density measure based on the crowding distance of particles (Fig. 2).

### 3 Simulation-based Optimisation Approach

In this research, the simulation model is conceptualized and developed following object-oriented principles and is implemented using the Microsoft Visual C-Sharp programming language which is one of several languages that support object-oriented programming. Traditional commercial discrete event simulation software presents two problems: (i) difficulties in modelling complex scenarios; (ii) too many entities could cause computational heavy simulation models (Cimino et al. 2010; Chatfield et al 2006). An object-oriented simulation system provides researchers with a flexible and extendible simulation tool designed to address the complexities in modelling supply chains. Biswas and Narahari (Object Oriented Modeling and Decision Support for Supply Chain, 2002) classified the elements of an object library of a simulation tool for supply chain networks into two categories: structural objects and policy objects. As the structural objects define the physical entities of networks, the policy objects describe the protocols used in procurement, manufacturing, transportation, and distribution of material within the supply chain. The current framework consists of various classes representing elements within a supply chain such as factory, warehouse, retailer, supplier, and customer.

The methodology used for solving the multiobjective inventory problem involves two phases as shown Figure 3 (Niranjan 2008):

- At the first phase, a multiobjective procedure based on MOPSO algorithm is used to determine both the reorder point and the order quantity of each stock point.
- At the second phase, the developed object-oriented simulator is used to evaluate the supply chain performance measures, according to different criteria: customer service level, fill rate, number of backordered items, and total cost.



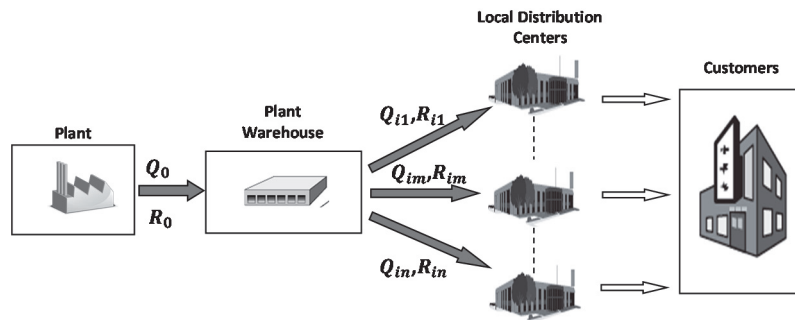
**Figure 3:** Simulation-based optimisation scheme based on MOPSO for the inventory problem

An individual in MOPSO algorithm is an array of inventory decision variables of the problem under study. In an n-facility supply chain problem, the decision variables

for the optimisation procedure include an order quantity vector  $[Q_1, Q_2, \dots, Q_n]$  and a reorder point vector  $[R_1, R_2, \dots, R_n]$ . The initial population is generated randomly based on the upper and lower bound for each of the decision variables using a uniform distribution  $U[R_i^{LB}, R_i^{UB}]$  and  $U[Q_i^{LB}, Q_i^{UB}]$ .

#### 4 Description of the Case Study and Computational Results

The research approach and modelling methodology was applied to the supply chain of a major food product company in Europe. The given network consists of 1 manufacturing sites, 1 plant warehouse, 19 regional distribution centres (DC), and approximately more than 1000 retailers spread over the country (Fig. 4). The regional distribution centres are located in the network to help for pool risk and consolidating shipments from the plant warehouses. At any moment of time, the manufacturing facility is idle and producing the given item with a capacitated production rate. Each distribution centre receives daily orders of items from retailers and customers. The average daily demand is fitted to some of the theoretical probability distributions using historical sales data of DCs.



**Figure 4:** Simulation of two-echelon divergent production-inventory model

The interrelationship between transportation cost, shipment sizes and distance adds another dimension of the complexity to incorporate the transportation cost into the inventory analysis. In this research, the distance dependent cost for a truck based on the full vehicle load is assumed to be  $5,46 \times (Distance)^{-0,278}$  vehicle-km (Janic, 2007). The LTL transportation cost rates offered by the transportation 3<sup>rd</sup> party for four major distances, which are approximately 100, 250, 500 and 1,000 km in length, is illustrated in Figure 5 (Aldarrat, 2007).

The run length of the simulation experiment is 365 days. The particle swarm optimisation parameters are as follows: the swarm size is 40, the maximum archive size is 40 and the number of iterations is 100. The distribution of the Pareto optimal set over the trade-off surface is shown in Figure 6. It can be seen that solutions are widely distributed over the Pareto-optimal front due to the diversity of the non-dominated solutions in the proposed simulation-based MOPSO technique and the problem under study is solved effectively. The non-dominated solution that represents the best cost with related example inventory parameters is given in Table

1. For example, the service level of 100% produced a daily cost of 7.549 € while 89% service level produced a daily cost of 6.827 €.

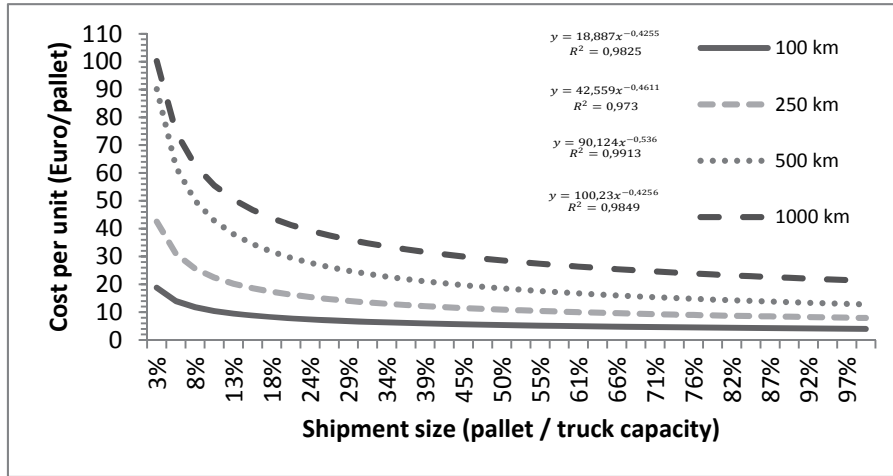


Figure 5: Examples of freight rates (distance-shipment based)

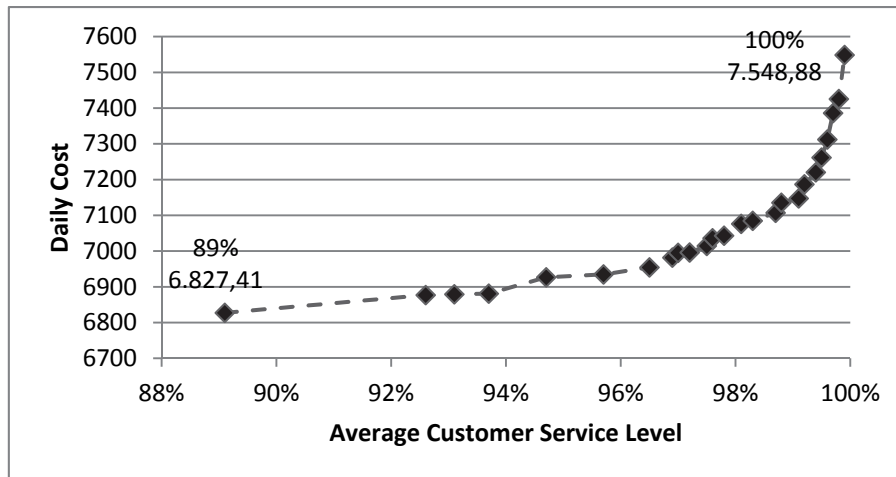


Figure 6: Final Pareto front of MOPSO-SO for the production-inventory system

**Table 1:** The non-dominated solution that represents the best cost with related example inventory parameters

	DC1	DC3	DC7	DC10	DC14	DC18
R	24	37	30	35	31	53
Q	75	81	109	75	148	137

## 5 Conclusion

This research introduces a simulation-based optimisation tool to analyse inventory control parameters in a multi-echelon production-inventory system under the continuous review policy in order to minimize the total system wide cost and maximize the customer service level. This study suggests the concept of MOPSO to find a set of near optimal solutions for multiple objectives. It has been shown that the MOPSO algorithm is a powerful, intelligent optimisation algorithm that is able to obtain non-dominated solutions of the multiobjective inventory problem. The problem can be extended in the future to include other inventory policies, different supply chain configurations, capacity limitations, uncertain costs, and raw material availabilities.

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