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Development of a Simheuristic Approach for Solving Realistic Inventory Routing Problems

Entwicklung eines Simheuristikansatzes zur Lösung von realitätsnahen Inventory-Routing-Problemen

Lena Pfeilsticker, Markus Rabe, TU Dortmund, Dortmund (Germany),
lena.pfeilsticker@tu-dortmund.de, markus.rabe@tu-dortmund.de

Angel A. Juan, IN3-Open University of Catalonia, Barcelona (Spain),
ajuanp@uoc.edu

Abstract: In order to stay or become even more cost-efficient, companies have to cooperate along the supply chain for improving the efficiency of transport and inventory systems. The majority of authors dealing with a problem in this research area either address the inventory problem or the transportation problem. Fewer published works deal with the integrated decision-making of inventory and transportation, called Inventory Routing Problem (IRP). In this paper, an approach is developed, that enables to find feasible low-cost solutions for complex IRPs in efficient time frames and under high levels of uncertainty. A suitable approach is to combine heuristic optimisation with simulation techniques, called 'simheuristics'. In this paper, a stochastic IRP with realistic work conditions is modelled and implemented. Furthermore, experiments are conducted, validating the implementation with an advantage in computation time and low costs in comparison to other conventional solving methods. The results are a significant contribution to create a link between companies and research in the field of IRPs.

1 Introduction

Within supply chains, independent decision-making processes are still a common approach. However, by cooperating along the supply chain, potentials to improve the transport and inventory systems can be exploited. If the supply chain were to be considered as a whole, an integrated decision-making could lead to significant decreases in total costs and operation durations. The general problem of integrating inventory and transportation (vehicle routing) decisions is called Inventory Routing Problem (IRP). Considering the reality of supply chain systems and real-life IRP applications, it is necessary to conduct research by considering the whole logistical system to find solutions with minimal cost and highest efficiency. A significant progress in this research field has already been reported by Juan et al. (2014a). The authors propose an approach combining simulation and heuristics to a so-called ‘simheuristic’ (Juan et al. 2015), leading to solutions which are improving known solutions to IRPs.

In this paper, a stochastic IRP is introduced. It is explained how an approach combining simulation and heuristics is able to find a good and feasible solution. Also, the paper demonstrates the implementation of the solution process as a Java application. Figure 1 shows an exemplary Inventory Routing System.

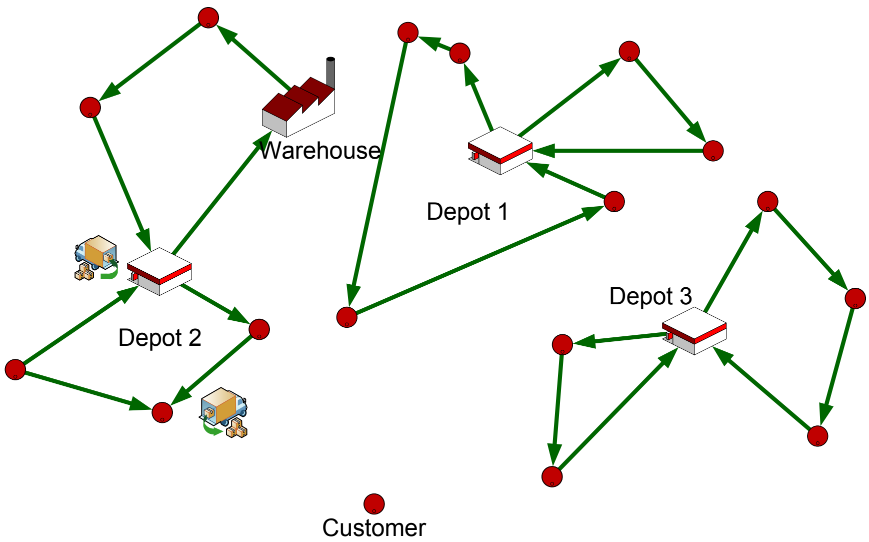

Figure 1: *Exemplary Inventory Routing System*

The paper is structured as follows: In Section 2, existing approaches to solve IRPs are discussed, with a special focus on the combination of simulation and heuristics. The methods additionally applied are introduced shortly. Section 3 introduces the given problem. Section 4 explains the proposed solution process and explains the benefit of the applied methods for solving stochastic IRPs. The implemented solution process is described in Section 5. For selected essential elements, the algorithm is explained. In Section 6, the paper evaluates several conducted experiments with the final implementation. Finally, the results are concluded in Section 7.

2 Simheuristics Approaches to Solve Inventory Routing Problems

Several studies have combined simulation and optimization approaches for solving complex optimization problems (van Dijk and van der Sluis 2008). The supply chain process has been a popular target for this type of techniques. In the past years, several approaches have been proposed for different variants of the IRP. The main factors to take into account when classifying the different works are: (a) whether they consider deterministic (Bertazzi et al. 2008) or stochastic demands (Federgruen and Zipkin 1984); (b) whether they consider single- (Powell and Godfrey 2002) or multiple-periods (including an infinite horizon) (Trudeau and Dror 1992); (c) whether they allow for inventory shortages (Jaillet et al. 1992); (d) whether they consider single (Campbell et al. 1998) or multiple products (Mjirda et al. 2014); (e) whether they use the same replenishment policy for all nodes (Bertazzi et al. 2013) or personalized replenishment policies for each node (Juan et al. 2014); and (f) whether they use exact (Baldacci et al. 2011) or approximate methods (Subramanian et al. 2012) to solve the problem.

In the present paper the solution process for the given problem is embedded in a simheuristic framework. It is able to build an interface between real-life stochastic IRPs and deterministic methods commonly used for complex problems in this research area (van Dijk and van der Sluis 2008). Simheuristics are hybrid simulation-optimization methods used for NP-hard, combinatorial optimization problems (COPs) with stochastic components (Juan et al. 2015). They allow for obtaining good feasible solutions in reasonable computing times. When applying a simheuristic approach, a stochastic COP is converted into a deterministic problem by using the expected mean values of the single input parameters. This problem is then solved using a heuristic approach and the deterministic solution is tested for feasibility. In a fast simulation process with a low number of replications, the promising solutions are evaluated by using stochastic parameters, leading to a stochastic solution to the COP. The generated solutions are ranked according to their best outcome and the best results are again tested for their feasibility and quality in an intensive simulation process with a large number of replications to determine the best solution (Juan et al. 2015). The method indirectly assumes that high-quality solutions for the deterministic version of the problem are likely to be high-quality solutions for the stochastic version. If this correlation is not perfect, it is possible to miss some good solutions for the stochastic version. The overall process is displayed in Figure 2.

During the process of fast simulation, the best solutions are stored in a cache. When the iterative process of finding solutions is repeated, the newly computed solutions are compared to the already computed best solutions in the cache. If the solutions in the cache are worse, the newly computed solution replaces the worst solution in the cache. Using this approach, promising solutions are selected over inferior ones. The approach does not guarantee finding the overall best solution for the problem it is applied to. But, it helps finding reasonably good solutions in a given maximum time.

During several steps of the solution process, a random sampling from skewed probability distributions is used in a Monte Carlo simulation. This approach is used to generate randomness in the heuristics (Juan et al. 2013). When applying a heuristic that is employed in a simulation process, the next constructive movement is selected based on a previously selected criterion. In the here treated IRP, the elements that can be chosen to construct the next movement are often sorted in a list from which they are selected. The order of this list is based on the benefit for the solution. E.g., for an IRP, a list of customers with the closest location to a depot is computed and the depot gets assigned to the customers with the top positions on the priority list, i.e. the customers with the minimal distance to the depot.

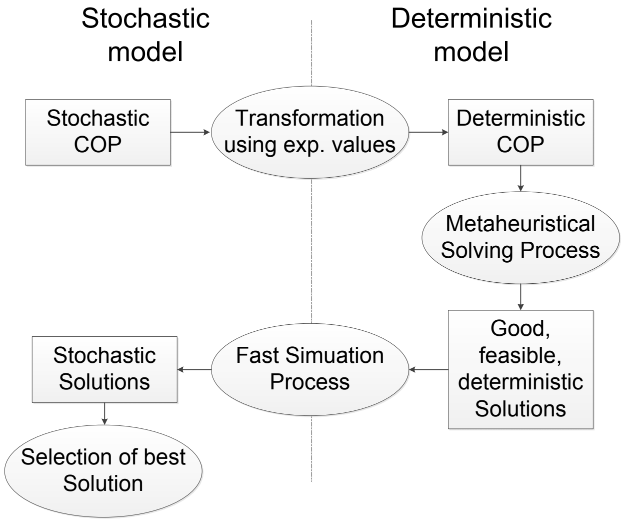

Figure 2: *Simheuristic approach (based on Juan et al. 2015)*

However, heuristics are deterministic approaches: using a specific input will always lead to the same results. To obtain several solutions from which the best solution can be selected, the elements of the priority list are combined with skewed distributions: A probability to be selected is matched with each element of the priority list, generating a stochastic approach. In the following simulation, the order in which the elements of the priority list are selected is different at every step of the iterative solution process and multiple solutions are obtained. A uniform distribution of the priority list of elements would destroy the logic of creating a priority list. Therefore, a skewed distribution has to be employed, assigning higher probabilities to be picked to elements higher on the priority list. This approach has been proven to outperform other common approaches used to solve this type of problems (Juan et al. 2014a). Their problem is described with the following conditions: single-period, single-product, single-depot, homogeneous fleet, centralized control unit, possible stockouts, stochastic demands and no time windows for the deliveries.

Further methods are the round-robin scheduling applied in the assignment of customers to depots by which they are served. A randomized variant of the Clarke and Wright savings (CWS) algorithm is used for determining the routing of vehicles from the depots to the assigned customers. An iterated local search (ILS) process is employed for refining solutions during a step of assigning customers to depots (Juan et al. 2014b).

3 The Given Inventory Routing Problem

The environmental work conditions for the problem in the present work are determined previously to the problem definition. For this purpose, an analysis of current literature and information provided by several companies has been conducted. Requirements for the treated problem should not be new to the field of IRPs but have been considered before in other research, to be sure that an impact on the problem exists. In order to achieve the goal to solve a realistic IRP, a significant number of working conditions or requirements (see below for some examples) have to be chosen, reflecting the reality of complex inventory routing systems. This is opposed to the approach of many other authors focusing on a low number of working conditions (van Dijk and van der Sluis 2008). In order to present a solution to a new IRP, the requirements are chosen in a combination that is not similar to characteristics in other authors' works. When choosing requirements, it is beneficial when the information provided by companies and the results from the literature research match. Finally, characteristics that are already used in Juan et al. (2014a) are extended and applied to the here treated IRP.

After reviewing all proposed characteristics and after a check for the accordance of them with all considered sources, a combination of several requirements is defined to reflect the reality of IRPs sufficiently while at the same time being implementable in a code. Based on the reviewed literature (see section 2) and the analyzed information provided by real-life companies, the requirements for the treated IRP are set as follows: It is considered with multiple products, multiple depots, one warehouse, a heterogeneous fleet, a single period, stochastic customer demands, a centralized control unit, possible stockouts and without time windows for the deliveries. The warehouse in this scenario is considered as a storage unit with virtually unlimited stock levels and can be accessed by vehicles sent from the depots. The solution comprises the completion of the delivery of all demanded products in a network of customers, depots and a warehouse, while minimizing the overall costs. The cost function consists of the set-up costs for the sending of one vehicle, of the inventory costs resulting from a stock cost rate and the number of stored products and a variable distance-based cost for travelled ways between nodes in the system. Thus, comparing with Juan et al (2014a), this research extends the problem to multiple products and depots as well as a heterogeneous fleet.

4 Proposed Solution Process

The solution process is divided into three steps, which are computed in a simheuristic framework: assignment, inventory planning and routing. The goal is to find the overall minimum costs. In this section, an overview is given on the process. The details are paid closer attention to during the implementation description.

The inputs for the computation of the solution are as follows: location of depots, location of warehouse, location of customers, stochastic customer demands, current inventory levels of every depot for every product, routing-costs function, inventory-costs function, number of vehicles by type and their associated number of compartments.

During the first step of the solution process, customers are bound to a specific depot. The result of this step is a single-depot IRP for each depot. A number of sub-maps is created, equal to the number of depots. This approach is based on a 'splitting policy' and decreases the problem complexity exponentially (Juan et al. 2014b). During this assignment step, a priority list of nodes is generated for each depot. The list is based on marginal savings in distance costs and considers the maximum capacity of the depots with regard to the combined customer demand for every product. To obtain multiple solutions for the comparison of the best outcome, i.e. lowest overall costs, a random sampling with a skewed distribution is introduced to assign customers to depots (cp. Section 2). Random sampling from skewed distributions is applicable whenever a new order of given elements needs to be constructed based on a sortable criterion during a Monte Carlo simulation, as described above. The order of the elements can be randomized and thus new solutions can be computed, based on a logical order. When solving an IRP, a lot of elements need to be arranged and skewed distributions favour the beneficial choices during the process. Then, a first assignment solution is generated using a round-robin process. At each round of the round-robin process the depot with the most (remaining) overall serving capacity selects a node from the generated list. After obtaining a first solution, a multi-start process generates thousands of maps with feasible solutions of assignment (customers to depot) in a short time period.

In the next step, the optimal inventory levels are estimated for each product of every depot, by considering the stochastic customer demands. For each of the depots, different replenishment policies are tested to determine the inventory levels with the lowest costs. For this purpose, the overall customers' demands of each depot are estimated. This is performed based on the results from the previous assignment step. Then, the expected inventory costs associated with each combination of depot and replenishment policy are calculated, including stockouts. With a basic CWS heuristic, the routing costs resulting from each replenishment policy are calculated. The policy with the lowest overall costs is chosen to serve for a computation of an initial base solution for the routing process.

The routing step generates routes from each depot to each of its assigned customers, employing a CWS heuristic with skewed distributed probabilities. Then, a first solution is computed. An unlimited number of the smallest trucks available is provided to form routes directly to and from the depots to every customer assigned to it. Then, the randomized CWS heuristic is performed to improve the first solution (Juan et al. 2011). While performing the CWS heuristic, the vehicle with the biggest surplus of overall capacity is chosen first, accounting for its number of different compartments and their individual capacity as opposed to the customers' demands. If a customer's number of demanded products is higher than the number of compartments in the biggest available truck, the customer is served by a minimum of two different trucks. The CWS heuristic chooses the next nodes based on their highest saving values. These values are the base of a priority list which is again randomized. Thus, a different outcome of routes generates thousands of feasible solutions in short computing times and the best solution found in a defined maximum time can be chosen. The result of the randomized CWS heuristic is improved with the help of a route cache. As described in section 2, this step is part of the intensive simulation process on elite solutions.

After the whole process, the solution with the overall lowest costs is returned as best solution. The outputs after the previously explained steps are the overall costs for routing and inventory, the set inventory levels with their associated replenishment policies and the routing plan for each depot. A flowchart of the proposed solution process is illustrated in Figure 3.

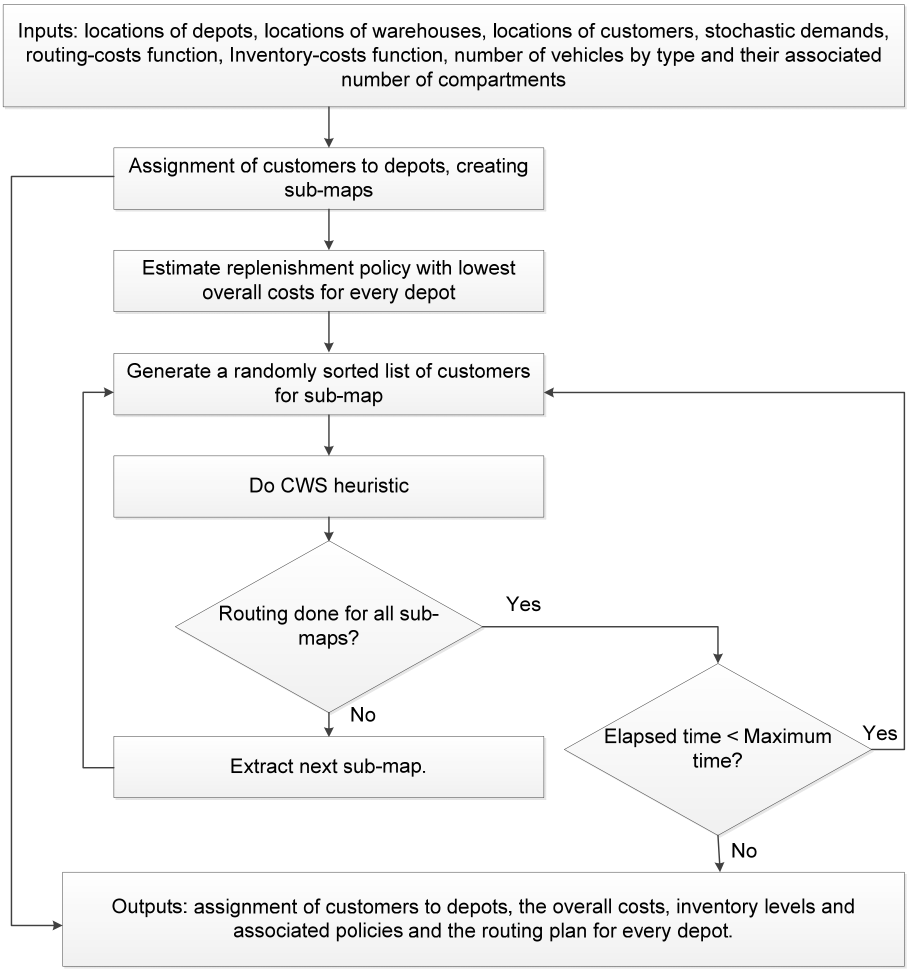

Figure 3: *Flowchart of the proposed solution process*

5 Implementation

The implementation is part of an iterative process. Since a lot of elements of the solution process are heuristic, there is no optimal solution and no reference values of comparable IRP solutions exist. Thus, to obtain a working code of the implemented solution process' utility, its development is conducted in several stages.

As in the proposed solution process, the customers are assigned to depots based on a capacitated round-robin scheduling during a multi-start approach. While assigning customers to the depots, the depot with the highest capacities left is chosen to be assigned to the next customer. The customers are chosen from a priority list based on marginal savings and with the help of oriented randomization. The distances between all customers, depots and warehouses are computed with coordinates and the Euclidean distance. Ensuing the assignment process, the comparison of inventto-ry policies leads to set inventory levels of each product for each depot. As described in the proposed solution process, an element is embedded in the code that introduces randomness, called a random number generator (RNG). This is used for the randomization in the assignment and routing steps of the algorithm. When merging the routes during the randomized CWS heuristic, all the vehicles available at one depot are ideally used to generate the highest savings possible. Thus, when merging two nodes, the new route with the highest savings value chooses an available vehicle with the highest surplus of capacity. This leads to assigning the biggest vehicles to the routes with the highest savings. Since the merging is conducted with the help of oriented randomization, the matching of saving values and size of vehicles is expected to have a reasonably good result. By using oriented randomization of the decisions about choosing a vehicle for a route, alternative scenarios are considered and the solution with the best result is returned as an output when running the code.

6 Experiments

The code was implemented in several stages. A basic structure with deterministic inputs was developed and finally, after a number of extensions, stochastic customer demands and calculations to consider those demands, when setting the inventory levels, were added.

Several experiments were performed, validating and verifying the code at each developed stage. The final implementation is constructed as described in the proposed solution process. It is easily and quickly possible to adapt the input values and probability distributions used to compute results. The algorithm is able to consider different replenishment policies $p = \{0.25, 0.5, 0.75, 1.0\}$ for every depot, indicating the percentage of refilled inventory (e.g. 0.5 = 50%) and to set the individual inventory levels according to the used probability distribution, based on the lowest-costs scenario. The code is also able to test different scenarios with varying fluctuation of customer demands while considering different stock cost rates. On the basis of the code output, a risk analysis can be conducted, allowing to evaluate the ensuing costs associated with different variances of customer demand.

In the following, an analysis of the final implementation outputs is conducted. 15 different scenarios were tested and compared regarding their outcome. The scenarios were based on different variances k in customer demands and different stock cost rates λ. The experiments were performed with three depots (D1, D2, D3).

The overall costs and the ratio between inventory and routing costs are displayed in Figure 4. It shows the overall costs of every combination of variance k and stock cost rate λ. The best computed policies p for every depot are listed below in Figure 4. The numbers 1 to 5 indicate the assigned possibility (1 = 0.0, 2 = 0.25, 3 = 0.5, 4 = 0.75, 5 = 1.0).

The results reflect the costs adequately: The higher the stock cost rate λ, the higher the overall inventory cost. Furthermore, as can be abstracted from Figure 4, in the case of $p = 0.0$ in all depots, no inventory costs are incurred or if one or two depots out of three have an inventory set at $p = 0.0$, the overall inventory costs are lower, respectively.

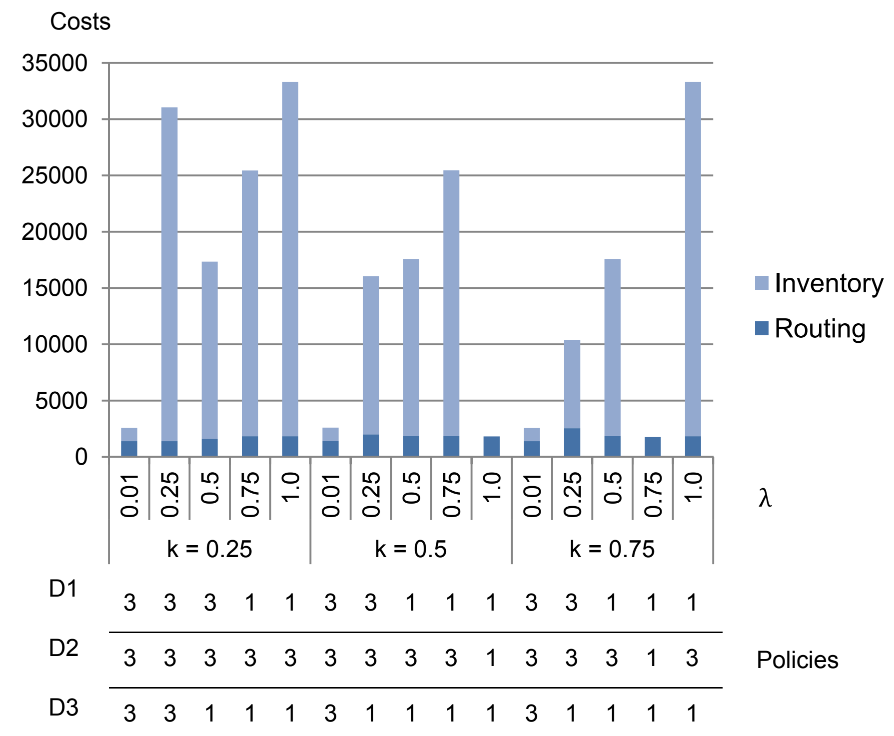

Figure 4: *Results of Stochastic Experiments*

7 Conclusion

In this research, a simheuristic algorithm was presented for solving a realistic rich single-period, multi-depot IRP with stochastic customer demands, stockouts and a heterogeneous fleet. The proposed methodology for the solving process combines several approaches to enable an integrated decision-making process for the simultaneous planning of inventory levels and routing. The research area of implementing rich IRPs is challenging because of highly complex requirements and their interdependencies, especially when introducing stochastic customer demands. Due to the combination of several methods, the developed program is able to deal with a high number of conditions in realistic IRPs under great levels of uncertainty. It can be used with any probability distribution to design realistic customer demands, is able to different replenishment policies for every depot and can test scenarios with different variances and ensuing costs thereby possessing the ability to analyze risks. The overall conclusion is that the program contributes significantly to the research field of IRPs due to its flexible applicability and possibility to be used on real-life cases.

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